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# Parametric Aerodynamic Design of Spinning Finned Projectiles Using a Matrix Interpolation Method

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**An inexpensive method suitable for numerical parametric studies is introduced. The computational approach has been applied to the preliminary aerodynamic design of high $L/D$ spinning finned projectiles. The influence of several geometric wing parameters on the basic aerodynamic coefficients has been investigated for a supersonic Mach number of 4. Results are provided to achieve the required design rapidly. Finally, some calculations show the precision of the matrix interpolation method.**

## Nomenclature

$A$ = input space point of computational program
$a_{ij}$ = coefficient model $ij$
$Ca$ = root chord
$Ce$ = tip chord
$Cl$ = roll moment coefficient
$Clp$ = roll damping moment coefficient
$Cm$ = pitching moment coefficient
$Cm_\alpha$ = slope of pitching moment coefficient
$Cmq$ = pitch damping moment coefficient
$CN$ = normal force coefficient
$CN_\alpha$ = slope of the normal force coefficient
$Cn$ = yaw force coefficient
$Cnp_\alpha$ = Magnus moment coefficient
$Cx_0$ = axial force coefficient
$Cy$ = side force coefficient
$Cyp_\alpha$ = Magnus force coefficient
$D$ = input space
$d$ = order polynomial function
$\ell_{i,j,k}$ = input variable $i, j, k$
$\ell_{i0}$ = center of the input variable region
$f_n$ = presumed functional relationship
$Ha$ = wing span
$j$ = polynomial degree (order)
$K$ = error parameter
$k$ = space dimension of input variables
$N$ = number of calculations
$p$ = space dimension of response
$R_{ij}$ = value of the response $j$ in the $i$th calculation
$R_{i,j,p}$ = response $i, j, p$
$S(x)$ = sum square of the response error
$S(\hat{x})$ = least sum square error response
var $S(x)$ = variance of the response
$v_{ij}$ = value of input data $j$ in the $i$th calculation
$X$ = input calculations matrix
$X'$ = input $X$ transposed
$X^{-1}$ = input $X$ inverse matrix
$Xcp$ = center of pressure
$Xcpm$ = center of pressure for the Magnus force
$x_i$ = coded input variable $i$
$\alpha$ = angle of incidence
$\beta$ = function's coefficient
$\delta_{\ell i}$ = input variable step
$\delta_i$ = range input variable
$\epsilon$ = response error
$\phi$ = roll position

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## Introduction

THIS paper describes a low-cost numerical method useful for parametric studies. The basic idea is to use several polynomial functions instead of a computer program. Each output parameter of a computer program could be associated with a polynomial function of the input parameters.

In the approach, a functional relationship between an output parameter and the input parameters is presumed. The technical problem is to obtain the set of the coefficients of the presumed functional relationship (polynomial expression) in an optimal way.

Practically, these coefficients are obtained by a least square method on the basis of $N$ different calculations achieved by the computer program.

The fundamental idea is to reduce the number of calculations without degrading the accuracy of the presumed relationship. We used a model-building approach[1-4] known by experimenters.

This methodology was successfully tested with five aerodynamic computer programs, essentially based on the component built-up approach.[5-18] For example, for a supersonic Mach number of 4, parametric aerodynamic preliminary design characteristics of a high $L/D$ spinning finned projectile are presented. Finally, to justify this approach, the accuracy of the method is evaluated.

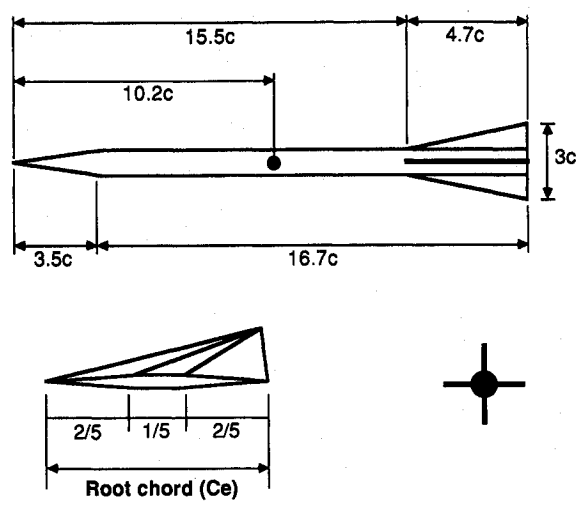


Fig. 1 Triangular projectile configurations.

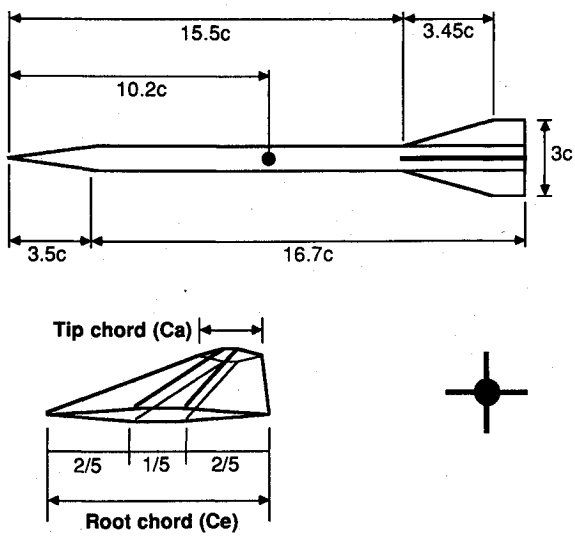


**Fig. 2 Trapezoid projectile configurations.**

## Analysis

### Numerical Method

A scientific computer program can be analyzed by two series of values: input and output data. The input variables are called prediction variables; they represent a space of dimension $k$: $\ell_1, \ell_2, \ldots, \ell_k$. These parameters are supposed to be quantitative and continuous. The output variables are named responses; they represent a space of dimension $p$: $R_1, R_2, \ldots, R_p$. The inputs for the computer program are a series of values given as the $\ell_i$ variables.

This program could 1) provide how a particular response $R_i$ is affected by a set of $\ell_i$ variables over some specific region of interest, 2) discover what setting values of the $\ell_i$ variables will give a product simultaneously satisfying specification for some responses $R_j$, and 3) explore the space of the $\ell_i$ variables to define the optimum response and to determine the nature of this optimum.

Often, the incremental approach consists in giving at each variable $\ell_i$ a variation step $\delta_{\ell i}$ and performing systematic calculations, step by step, to investigate the complete space of the $\ell_i$ variables.

All of the calculations can be represented by an $X$ matrix as follows:

$$\begin{array}{c} \\ 1 \\ : \\ i \\ : \\ : \\ N \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & \cdots\cdots\cdots & & j & \cdots\cdots & & k \end{array} \\ \begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & \cdots & \cdots & v_{1k} \\ v_{21} & v_{22} & v_{23} & \cdots & \cdots & \cdots & v_{2k} \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \boxed{v_{ij}} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ v_{N1} & v_{N2} & v_{N3} & \cdots & \cdots & \cdots & v_{Nk} \end{bmatrix} \\ \begin{array}{ccccccc} \ell_1 & \ell_2 & \ell_3 & \cdots & \cdots & \cdots & \ell_k \end{array} \end{array} = X \qquad (1)$$

where $v_{ij}$ is the value given to the $i$th calculations at $\ell_j$.

The size of the $X$ input matrix becomes increasingly important with the $k$ number of $\ell_i$ variables, and also with the size of the space of the $\ell_i$ variables. Therefore, the cost of the computation is important. When it is possible, the better approach is to postulate a presumed functional relationship:

$$R_j = f_n(\ell_i) \qquad (2)$$

The difficulties are the choice of the form of the $f_n$ function to fit $f_n$ and to analyze if the presumed $f_n$ function is representative of the physics of a specific problem. Box and Draper[1] suggest for routine experimental applications to postulate for $f_n$ a polynomial form in $\ell_i, \ldots, \ell_k$, which is a linear aggregate (or combination) of powers and products of the $\ell$.

A term in the polynomial is said to be of order $j$ (or degree $j$) if it contains the product of $j$ of the $\ell$ (some of which may be repeated). An $f_n$ polynomial function is said to be of order $d$ if the term(s) of highest order in it is (are) of order $d$. It is convenient to work with coded or standardized variables in input $x_i$. For example, if at some stage of an investigation we defined the current region of input interest for $\ell_i$ to be $\ell_{i0} \pm \delta_i$, where $\ell_{i0}$ is the center of the region, then it would be convenient to define an equivalent working variable $x_i$ where

$$x_i = \frac{\ell_i - \ell_{i0}}{\delta_i} \qquad (3)$$

The presumed $f_n$ function can be written if $n = 2$:

$$f_n = \beta_0 + (\beta_1 x_1 + \beta_2 x_2) + (\beta_1^1 x_1^2 + \beta_2^2 x_2^2 + \beta_{12} x_1 x_2) + (\beta_1^{11} x_1^3 + \beta_2^{22} x_2^3 + \beta_{12}^1 x_1^2 x_2 + \beta_{12}^2 x_1 x_2^2) + \text{etc.} \ldots \qquad (4)$$

In Eq. (4) the $\beta$ are coefficients that, in practice, have to be estimated from the $X$ matrix ($N$ calculations).

The number of such coefficients increases rapidly as the number $n$ of the input variables and the degree $d$ of the polynomial both increase. We must know the physics of the problem to make use of the graduating function $f_k$. A polynomial expression of degree $d$ can be thought of as a Taylor's series expansion of the true underlying theoretical function $f_n$ truncated after terms of $d$th order.

The following will usually be true[1]:

1) The higher the degree of the approximation function, the more closely the Taylor series can approximate the true function.

2) The smaller the region $D$ of the input space over which the approximation needs to be made, the better is the approximation possible with a polynomial function of given degree.

For fitting $f_k$ function from the $X$ matrix input data, we employed the least squares method. Since matrices are necessary to develop this method efficiently, the required matrix algebra is provided.[1,2] The interest of the methodology proposed in these references is to obtain the minimal length of the $X$ matrix ($N$ minimal) for the best fit of the $f_k$ function.

For the response function $R_j$, we have in matrix form:

$$R_j = Xx + \epsilon \qquad (5)$$

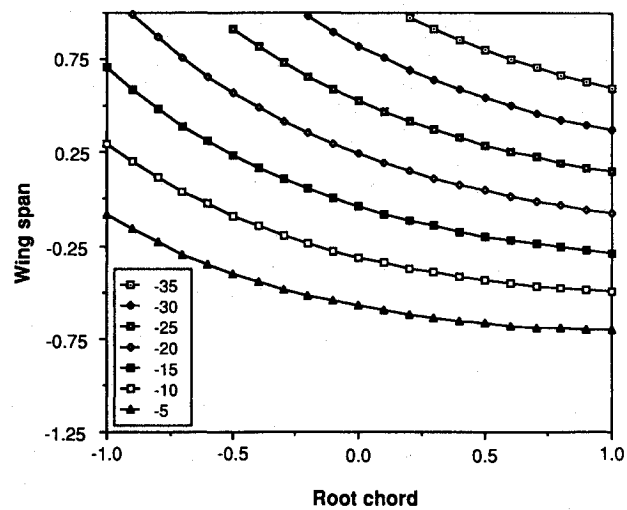


**Fig. 3 $Cm_\alpha$ contours (triangular).**

where

$$\begin{bmatrix} R_{1j} \\ R_{2j} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ R_{Nj} \end{bmatrix} = \begin{bmatrix} \nu_{11} & \nu_{12} & \cdots & \cdots & \cdots & \nu_{1k} \\ \nu_{21} & \nu_{22} & \cdots & \cdots & \cdots & \nu_{2k} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \nu_{N1} & \nu_{N2} & \cdots & \cdots & \cdots & \nu_{Nk} \end{bmatrix} \begin{bmatrix} 1 \\ x_{1j} \\ x_{2j} \\ \cdots \\ x_{1j}^2 \\ x_{2j}^2 \\ \cdots \\ x_{1j}x_{2j} \\ x_{1j}x_{3j} \\ \cdots\cdots \end{bmatrix} + \begin{bmatrix} \epsilon_{1j} \\ \epsilon_{2j} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \epsilon_{Nj} \end{bmatrix}$$

$N*1 \qquad N*k \qquad k*1 \qquad N*1$

The dimensions of these vectors and matrices are given beneath them, and $\epsilon$ represents the error.

The sum of squares is

$$S(x) = \sum_{i=1}^{N} \left[R_{ij} - f_n(x)\right]^2 \tag{6}$$

In matrix form, Eq. (6) can be written as

$$S(x) = (R_j - Xx)'(R_j - Xx) \tag{7}$$

The minimizing choice of $x$, its estimated least square, is denoted by $\hat{x}$; the corresponding (minimized) value of $S(x)$ is called $S(\hat{x})$.

If $A$ is a point of the input space of dimension $k$

$$A = \begin{bmatrix} x_{1A} \\ x_{2A} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ x_{kA} \end{bmatrix}, \qquad A_{fk} = \begin{bmatrix} x_{1A} \\ x_{2A} \\ \cdots \\ \cdots \\ x_{KA} \\ x_{1A}^2 \\ \cdots \\ \cdots \\ x_{kA}^2 \\ x_{1A}x_{2A} \\ \cdots\cdots \end{bmatrix}$$

The variance of the estimation of the response $R_j$ at the point $A$ is[2]

$$\mathrm{var}R_j(A) = KA'_{fk}(X'X)^{-1}A_{fk} \tag{8}$$

where $K$ is an error parameter that includes numerical effect. Good methodology is to choose the $X$ matrix to minimize $\mathrm{var}R_j(A)$ and $N$ in Eq. (8). One criterion is to choose the $X$ matrix as $D$ optimum.[1,2]

In this paper, for the numerical aerodynamic parametric study, a second-order model is used, and in order to fit the $f_k$ function, a Doelhert[3,4] $X$ matrix is used because of its low cost in calculations. With a Doelhert[3,4] for $k$ input variables, the number $N$ of calculations for fitting the $f_k$ function is equal to $k^2+k+1$. Generally, for investigating the input space of interest, we must give to each $\ell_{0i}$ variable a minimum of three values (with only two values, the analysis of the physical problem will be linear). Under these conditions, the number of calculations is $N=3^k$. Using a Doelhert[3,4] $X$ matrix, the difference is equal to $3^k-k^2-k-1$, which represents 60 calculation steps for four input variables.

**Table 1 Flight conditions (reference length is the body length)**

| | |
|---|---|
| Altitude | 0.00 m |
| Pressure | 1013.25 mb |
| Temperature | 288.16 K |
| Initial roll rate | 8377.58 rad·s$^{-1}$ |
| Mach number | 4.00 |
| Reynolds number | 14.90 (*10$^6$) |

**Table 2 Triangular wing plan study**

| | |
|---|---|
| Parametric input variables | $f_1 = Ca$<br>$f_2 = Ha$ |
| Region of interest | 0.0200 m $\le Ca \le$ 0.0370 m<br>0.0037 m $\le Ha \le$ 0.0083 m |
| Principal output variables (responses) | $R_1 = Cm_\alpha$<br>$R_2 = Cx_0$<br>$R_3 = Cyp_\alpha$ |
| Presumed functional relationship | $R_i = a_0^i + a_1^i x_1 + a_2^i x_2 + a_{11}^i x_1^2 + a_{22}^i x_2^2 + a_{12}^i x_1 x_2$ |

**Table 3 Triangular wing matrix: Doelhert[3,4] $X$ matrix, $N = 7$**

| | $Ca$, m*10$^{-3}$ | $(x_1)$ | $Ha$, m*10$^{-3}$ | $(x_2)$ |
|---|---|---|---|---|
| $X =$ | 37.500 | ( + 1.0000000) | 6.000 | ( + 0.0000000) |
| | 20.000 | ( − 1.0000000) | 6.000 | ( + 0.0000004) |
| | 33.125 | ( + 0.5000000) | 8.000 | ( + 0.8660254) |
| | 24.375 | ( − 0.5000000) | 4.000 | ( − 0.8660254) |
| | 33.125 | ( + 0.5000000) | 4.000 | ( − 0.8660254) |
| | 24.375 | ( − 0.5000000) | 8.000 | ( + 0.8660254) |
| | 28.750 | ( + 0.0000000) | 6.000 | ( + 0.0000000) |

**Table 4 Trapezoid wing plan study**

| | |
|---|---|
| Parametric input variables | $f_1 = Ca$<br>$f_2 = Ha$<br>$f_3 = Ce$ |
| Region of interest | 0.0200 m $\le Ca \le$ 0.0370 m<br>0.0037 m $\le Ha \le$ 0.0083 m<br>0.0011 m $\le Ce \le$ 0.0109 m |
| Principal output variables (responses) | $R_1 = Cm_\alpha$<br>$R_2 = Cx_0$<br>$R_3 = Cyp_\alpha$ |
| Presumed functional relationship | $R_i = a_0^i + a_1^i x_1 + a_2^i x_2 + a_3^i x_3 + a_{11}^i x_1^2 + a_{22}^i x_2^2 + a_{33}^i x_3^2 + a_{12}^i x_1 x_2 + a_{13}^i x_1 x_3 + a_{23}^i x_2 x_3$ |

### Computational Program Description

The aerodynamic programs used a component buildup approach. These programs are a reliable and low-cost tool for the preliminary design of a conventional geometric configuration.

1) The MISSILE code is based on the equivalent angle-of-attack concept of Nielsen.[5-7] This computer program has been developed by ONERA[8] and GIAT Industries for the specific problem of multifinned projectiles (number of wings per tail > 4). For a nonspinning fin-stabilized projectile, the MISSILE code gives $CN$, $CN\alpha$, $Cy$, $Cm$, $Cm_\alpha$, $Cl$, $Cn$, $Cmq$, and $X$cp. The flight conditions are Mach number, Reynolds number, $\alpha$, and $\phi$.

2) The SATIN code[9-13] is a computer program developed by GIAT Industries. This program gives primarily the axial force coefficient $Cx_0$.

3) The AUBE code,[9-14] developed by GIAT Industries, gives the wing contributions to the $Clp$, $Cyp_\alpha$, and $Cnp_\alpha$ coefficients.

4) The SPIN code[15] is a program for spinning bodies. Essentially, it provides $Clp$, $Cyp_\alpha$, $Cnp_\alpha$, and $X$cpm.

5) The CALSTA code[16-18] is a computer program developed by GIAT Industries for the stability analysis of angular motion of rolling projectiles.

### Projectile Configurations

The basic dimensions of the finned body configuration, for which calculations were performed, is presented in Figs. 1 and 2. The modeled finned body is characterized by a conical nose

**Table 5 Trapezoid wing matrix: Doelhert[3,4] $X$ matrix, $N = 13$**

$X =$

| $Ca$, m*10$^{-3}$ | ($x_1$) | $Ha$, m*10$^{-3}$ | ($x_2$) | $Ce$, m*10$^{-3}$ | ($x_3$) |
|---|---|---|---|---|---|
| 37.500 | ( + 1.0000000) | 6.000 | ( + 0.0000000) | 6.000 | ( + 0.0000000) |
| 20.000 | ( − 1.0000000) | 6.000 | ( + 0.0000000) | 6.000 | ( + 0.0000000) |
| 33.125 | ( + 0.5000000) | 8.000 | ( + 0.8660254) | 6.000 | ( + 0.0000000) |
| 24.375 | ( − 0.5000000) | 4.000 | ( − 0.8660254) | 6.000 | ( + 0.0000000) |
| 33.125 | ( + 0.5000000) | 4.000 | ( − 0.8660254) | 6.000 | ( + 0.0000000) |
| 24.375 | ( − 0.5000000) | 8.000 | ( + 0.8660254) | 6.000 | ( + 0.0000000) |
| 33.125 | ( + 0.5000000) | 6.700 | ( + 0.2886751) | 10.000 | ( + 0.8164966) |
| 24.375 | ( − 0.5000000) | 5.300 | ( − 0.2886751) | 2.000 | ( − 0.8164966) |
| 33.125 | ( + 0.5000000) | 5.300 | ( − 0.2886751) | 2.000 | ( − 0.8164966) |
| 28.750 | ( + 0.0000000) | 7.300 | ( + 0.5773503) | 2.000 | ( − 0.8164966) |
| 24.375 | ( − 0.5000000) | 6.700 | ( + 0.2886751) | 10.000 | ( + 0.8164966) |
| 28.750 | ( + 0.0000000) | 4.700 | ( + 0.8660254) | 10.000 | ( + 0.8164966) |
| 28.750 | ( + 0.0000000) | 6.000 | ( + 0.0000000) | 6.000 | ( + 0.0000000) |

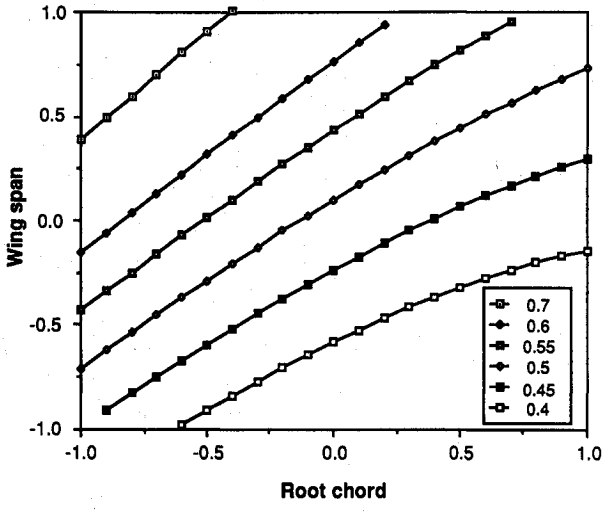


Fig. 4 $Cx_0$ contours (triangular).

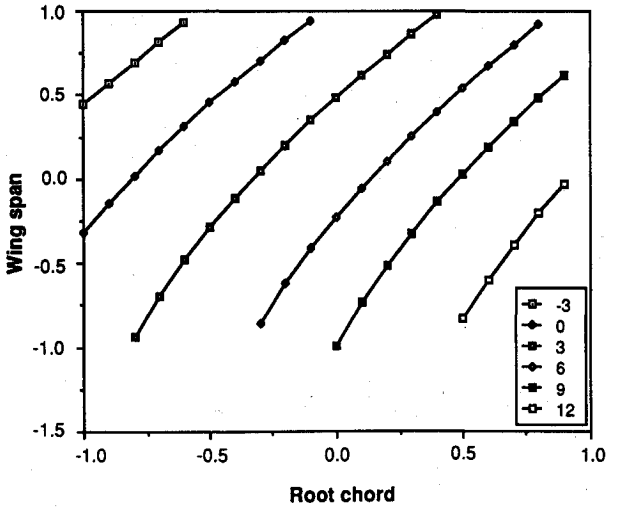


Fig. 5 $Cyp_\alpha$ contours (triangular).

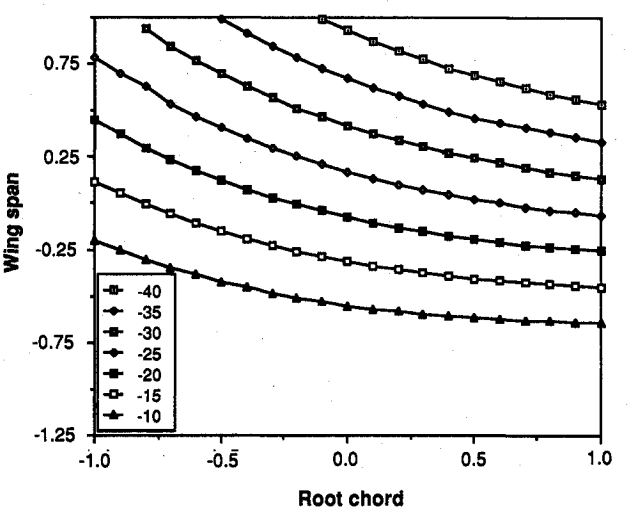


Fig. 6 $Cm_\alpha$ contours (trapezoid): $x_3 = 0.0$; $Ce = 0.75$ cal.

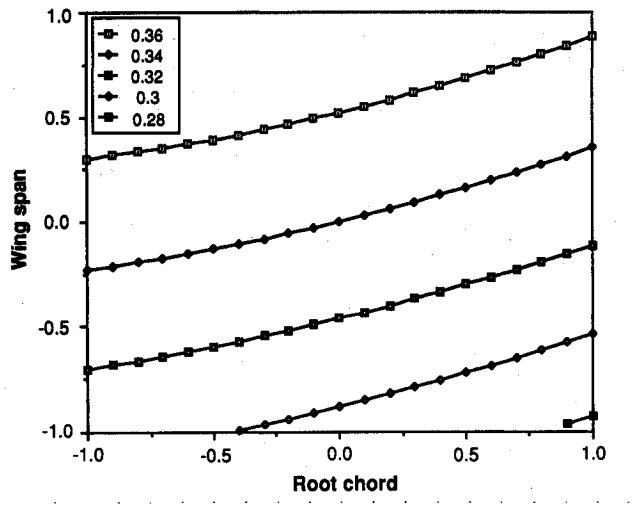


Fig. 7 $Cx_0$ contours (trapezoid): $x_3 = 0.0$; $Ce = 0.75$ cal.

that is joined to a smooth cylindrical main body with four symmetrical fins attached to the cylindrical aft section of the projectile. The swept fins are triangular or trapezoid, and the basic wing airfoil section is hexagonal.

**Parametric Study**

The flight conditions (see Table 1) and the body geometric characteristics are constant. The maximum thickness of the wing airfoil section is constant. Tables 2–5 give the characteristics of the parametrical study and of the Doelhert[3,4] $X$ matrix.

If $R_{ij}$ is the numerical yield in the $i$th calculation, least square estimates of $a_0^j, a_1^j, a_2^j, \ldots, a_{23}^j$ are obtained by the procedure defined by the matrix equation:

$$x = (X'X)^{-1}X'R_j$$

$$R_j = \begin{bmatrix} R_{1j} \\ R_{2j} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ R_{Nj} \end{bmatrix}, \qquad x = \begin{bmatrix} a_0^j \\ a_1^j \\ \cdots \\ a_N^j \\ \cdots \\ \cdots \\ a_{23}^j \end{bmatrix}$$

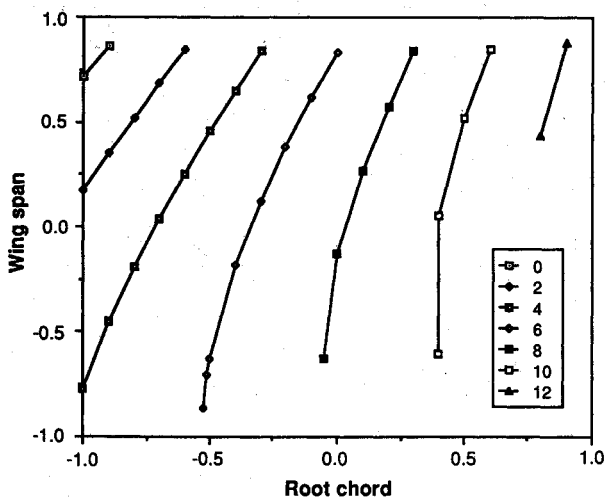


**Fig. 8** $Cyp_\alpha$ **contours (trapezoid):** $x_3 = 0.0$; $Ce = 0.75$ **cal.**

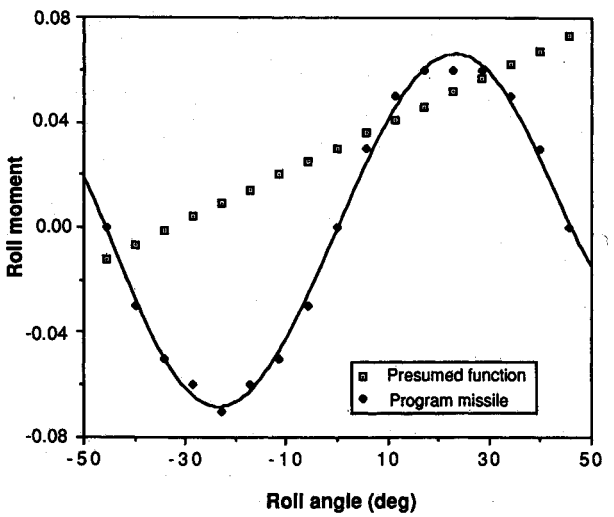


**Fig. 9** $Cl$ **vs** $\phi$ **for presumed function and computer program MISSILE.**

**Table 6 Model coefficients for triangular wings**

| Coefficient | $Cm_\alpha$ | $Cx_0$ | $Cyp_\alpha$ |
|---|---|---|---|
| $a_0$ | −15.709 | 0.485 | 5.129 |
| $a_1$ | −7.752 | −0.106 | 6.512 |
| $a_2$ | −18.094 | 0.148 | −4.003 |
| $a_{11}$ | 1.840 | 0.037 | 0.236 |
| $a_{22}$ | 0.881 | 0.003 | −0.791 |
| $a_{12}$ | −4.883 | −0.035 | −0.177 |

**Table 7 Model coefficients for trapezoid wings**

| Coefficient | $Cm_\alpha$ | $Cx_0$ | $Cyp_\alpha$ |
|---|---|---|---|
| $a_0$ | −21.555 | 0.340 | 7.847 |
| $a_1$ | −6.720 | −0.012 | 5.441 |
| $a_2$ | −20.574 | 0.041 | −1.306 |
| $a_3$ | −3.821 | −0.011 | 4.542 |
| $a_{11}$ | 1.680 | −0.003 | 0.117 |
| $a_{22}$ | 0.787 | −0.005 | −1.083 |
| $a_{33}$ | 0.376 | 0.006 | −0.471 |
| $a_{12}$ | −4.936 | 0.003 | 1.441 |
| $a_{13}$ | 0.590 | 0.012 | 2.101 |
| $a_{23}$ | −1.760 | −0.016 | 0.662 |

**Table 8 Regression analysis for triangular wings**

| Coefficient | $R^2$ | $\sigma$ |
|---|---|---|
| $Cm_\alpha$ | 0.9995 | 0.76996 |
| $Cx_0$ | 0.9931 | 0.02650 |
| $Cyp_\alpha$ | 0.9931 | 1.10710 |

**Table 9 Regression analysis for trapezoid wings**

| Coefficient | $R^2$ | $\sigma$ |
|---|---|---|
| $Cm_\alpha$ | 0.9998 | 0.32722 |
| $Cx_0$ | 0.9961 | 0.00322 |
| $Cyp_\alpha$ | 0.9768 | 1.30814 |

## Results

**Determination of the Presumed Functional Relationship**

Tables 6 and 7 give the coefficients of the presumed functional relationship calculated by the least square method.

**Global Effect of Standardized Input Variables on Responses in the Region of Interest**

The $x$ standardized input variables have no dimension and can be directly compared. A coefficient of presumed function represents the global weight in the region of interest of the standardized associated variable. As an example, the weight of the first interaction term $x_1x_2$ is $a_{12}^j$, etc.

**Contour Evolutions**

*Triangular Wing*

Using the coefficients of Table 6, Figs. 3–5 present, respectively, the evolution of the $Cm_\alpha$, $Cx_0$, and $Cyp_\alpha$ contours as a function of root chord and wing span. Figures 3–5 show easily that finding a geometric configuration is a compromise; the best geometric configuration for a finned spinning projectile is obtained for $|Cm_\alpha|$ maximum and $Cm_\alpha$ negative with $Cx_0$ and $Cyp_\alpha$ minimum. In the domain of interest, good stability ($Cm_\alpha < 0$) is obtained by increasing the wing span and root chord, but if the wing span increases, $Cx_0$ also increases (Fig. 4), and if the root chord increases, $Cyp_\alpha$ also increases (Fig. 5).

*Trapezoid Wing*

Figures 6–8 show the evolution of the $Cm_\alpha$, $Cx_0$, and $Cyp_\alpha$ contours as a function of the root chord and the wing span for a fixed value of the tip chord.

## Accuracy of the Numerical Method

The objective is to analyze the differences between real computer programs of aerodynamics and stability, with presumed functions fitted in the region of interest.

### Characteristics of Least Square Method

Tables 8 and 9 show the $R^2$ statistic multiple regression coefficients and the standard deviations of the regression for all of the fitted functions.

### Additional Systematic Comparison

As an example, in the case of a trapezoid wing, a random number generator with uniform distribution is used in order to define 20 triplets ($Ca$, $Ha$, $Ce$) in the region of interest (Table 4). First, the aerodynamic characteristics of each triplet are calculated with the MISSILE[8] computer program; in a second stage, the same coefficients are estimated with the presumed function obtained by the Doelhert[3,4] $X$ matrix. Comparison of the difference between the real computer program and the presumed function for the $Cm_\alpha$ prediction indicates accuracy of the presumed function within 6% of the computer program.

## Limitation of the Numerical Method

The main difficulty of this method is to choose the appropriate form of the presumed function $f_n$; $f_n$ must be representative of the physics of a specific problem. For example, a Doelhert[3,4] $X$ matrix is used to analyze the evolution of the roll moment coefficient vs the roll angle position of the trapezoid projectile configuration depicted in Fig. 2. The presumed function $f_n$ is quadratic. The Doelhert[3,4] $X$ matrix for five input parameters ($Ca$, $Ha$, $Ce$, $\phi$, $\alpha$) was obtained. Figure 9 shows the evolution of the roll moment coefficient $Cl$ vs the roll angle obtained by the presumed function and directly calculated by the computer program MISSILE.[6] The method employed is inaccurate for representing a well-known sinusoidal evolution of $Cl$ vs $\phi$. The quadratic $f_n$ form is inadequate. For this problem, it will be necessary to assume another $f_n$ function, which could include the sinusoidal effect of the roll angle.

## Conclusions

In the present paper, the numerical methodology using matrix fitting procedures was shown to be inexpensive, accurate, and suited for the determination of preliminary aerodynamic design. Employed for experimental studies, the model-building approach (or experimental design) seems to be useful for many computational problems. However, some limitations of the numerical methodology are shown.

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